An isomorphism theorem for ordered sets under Antiorders

Daniel A. Romano
University of Banja Luka
Bosnia and Hercegovina

Let (X,=,≠) be a set with apartness. A relation $\alpha \subseteq$ X×X is an antiorder relation on X iff

$$\alpha \subseteq \neq,\ \alpha \subseteq \alpha^{*}\alpha,\ \neq \subseteq \alpha \cup \alpha^{-1}.$$

A relation s on a ordered set (X,=,≠,α) under an antiorder α is a quasi-antiorder relation on X iff

$$s \subseteq \alpha \text{ and } s \subseteq s^{*}s.$$

Sometime, in the definition of antiorder relation on set (X,=,≠), we add an another condition $\alpha \cap \alpha^{-1} = \emptyset$. In that case, in the definition of quasi-antiorder relation on the ordered set (X,=,≠,α) under the antiorder α, we must add the following condition $s \cap s^{-1} = \emptyset$. In this short note we proved some kind of isomorphism theorem for ordered sets under antiorders. Let (X,$=_X$,$\neq_X$,α) and (Y,$=_Y$,$\neq_Y$,β) be ordered sets under antiorders, where the apartness $\neq_Y$ is tight. If φ : X $\rightarrow$ Y is reverse isotone strongly extensional function, then there exists a strongly extensional and embedding reverse isotone bijection ((X, $=_X$, $\neq_X$, α, c(R))/q, $=_1$, $\neq_1$, γ) $\rightarrow$ (Im(φ),$=_Y$,$\neq_Y$,β) where c(R) is the biggest quasi-antiorder relation on X under R = $\alpha \cap$Coker(φ), q = c(R)$\cup$c(R)$^{-1}$ and γ antiorder induced by the quasi-antiorder c(R). If the condition $\alpha \cap \alpha^{-1} = \emptyset$ holds, then the above bijection is the isomorphism

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